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THE INFLUENCE OF ANHARMONIC CURRENT-PHASE RELATION ON PENETRATION DEPTH IN LONG JOSEPHSON JUNCTION

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Abstract. We study the long Josephson junction model, which takes into account the second harmonic in current-phase relation. We use the current-phase relation with second harmonic $\sin 2\phi$ in addition to the first harmonic $\sin\phi$. The influence of the second harmonic on the stability of magnetic flux distributions for main solutions is discussed. The influence of the second harmonic on Josephson penetration depth was calculated using the expression for effective critical current. Obtained results compared with results of another theoretical models.

Keywords: *Josephson junction, current-phase realtion, sine-Gordon equation.*

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1. Introduction

Low dimensional structures play important role in many electronic devices. Optimization of their properties and functioning of such devices require tuning the material properties and revealing the most appropriate device architecture. This concerns also Josephson junctions. Physical properties of magnetic flux in Josephson junctions play important role in the applied superconductivity. The remarkable feature of Josephson junctions is the fact that the phase difference at the junction is described in terms of the sine-Gordon equation (Barone *et al.*, 1982; Likharev, 1986; Askerzade *et al.*, 2017). In the investigations of the dynamics of such structures, the current-phase relation of Josephson junction (Askerzade *et al*., 2017).

$$I = I_c \sin\phi \tag{1}$$

were used. The relationship (1) is fulfilled with high accuracy for Josephson junctions on low-temperature superconductors (Il'ichev *et al*., 2017). In the case of junctions on high-temperature superconductors, the current-phase relation becomes anharmonic (Tsuei *et al*., 2000).

$$I = I_{c0} f_\alpha(\phi) = I_{c0}(\sin\phi + \alpha\sin 2\phi), \tag{2}$$

where anharmonicity parameter α depends on the junction preparation technology. In general, anharmonicity in the current-phase relation for high-temperature and Fe-based superconductors based junctions are associated with the d-wave behavior of the order parameter and many band characters of superconducting state in new superconducting compounds (Askerzade *et al*., 2012). Dynamical properties of single junctions with an anharmonic current-phase relation (2) were previously studied in papers (Canturk *et al*., 2011; Canturk *et al*., 2012).

In the case of long Josephson junctions with anharmonic current-phase relation, the phase dynamics and the influence of fluctuations effects can exhibit new features. Especially, the presence of second harmonic (Eq. (2)) can lead to significant changes of shape and stability properties using numerical methods (Atanasova *et al*., 2010; Atanasova *et al*., 2011; Dimov *et al*., 2019). In this study, we discuss the soliton dynamics of long Josephson junction with anharmonic current-phase relation. The details of the changing of London penetration of junctions with unconventional current-phase relation in single long Josephson junction are analyzed using effective critical current in such juncrion.

2. Basic equations

The physics of solitons in long Josephson junctions appears in various contexts within nonlinear physics, superconductivity, and high-frequency device applications. A soliton in a long Josephson junction is often called a "fluxon" since it accounts for a magnetic flux quantum $\Phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15}$ Wb moving between two superconducting electrodes of the junction. A fluxon in a long Josephson junction carries a magnetic flux equal to one flux quantum. Its appearance is explained in Fig. 1. This figure shows the cross-view of the junction in the plane perpendicular to the external magnetic field H. Josephson tunnel barrier is a thin (1 - 2 nm thick) layer of insulator (I) between two superconducting electrodes (S). Due to the Meissner effect, the external field is screened by circulating supercurrents and it penetrates inside a bulk superconductor to the distance known as the London penetration depth λ_L (Schmidt, 1997; Ustinov, 2008).

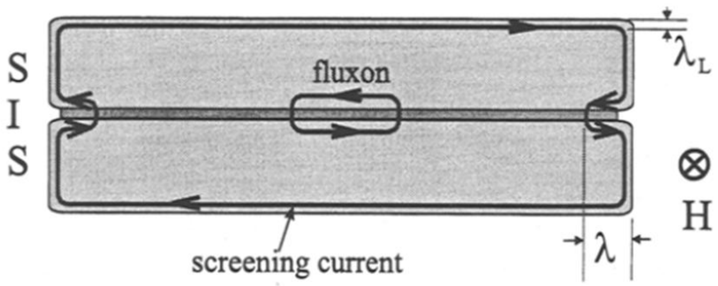

Fig. 1. Schematic view of long Josephson junction

In the region of the Josephson barrier, the screening effect is weakened, thus the magnetic field penetration distance is larger. This distance is called the Josephson penetration depth λ. Its value depends on the strength of the Josephson coupling between electrodes. The screening currents form a "tangle" penetrating to the distance of about λ, into the Josephson junction.

Mathematically, the fluxon corresponds to a *2π* kink of the quantum-mechanical phase difference c *2π* between the two superconducting electrodes of the junction. The sine-Gordon equation which describes the quasi-one-dimensional dynamics of the system, in normalized form, is (Schmidt, 1997; Ustinov, 2008).

$$\frac{d^2\phi}{dx^2} = \frac{1}{\lambda^2}\sin\phi \quad (3)$$

where London penetration depth is determined as

$$\lambda = \left(\frac{\Phi_0}{8\pi^2 j_c d}\right)^{1/2}. \quad (4)$$

In the last expression j_c critical current density of the Josephson junction, $d = 2\lambda_L + t$, t is the thickness of the insulating layer. To account for the behavior of a real junction, Eq. (3) must be solved together with the appropriate boundary conditions which depend on the junction geometry and take into account the magnetic field applied in the plane of the junction (Kuplevakhsky, 2006; Kuplevakhsky, 2007). Application of similar Eqs. for the study of soliton dynamics in branched Josephson junctions can be found in Ref. (Sabirov *et al*.,2018; Sabirov *et al*., 2020), An important solution to Eq. (3) is the soliton

$$\phi(x) = 4\arctan(\exp(\frac{x}{\lambda})). \quad (5)$$

This solution is shown in Fig. 2. It describes a 2π -kink moving with a velocity u and located at $x = x_0$ for $t = 0$. Equation (3) for both open and periodic boundary conditions has been discussed in detail using method of Sturm-Louville by in (Ustinov, 2008).

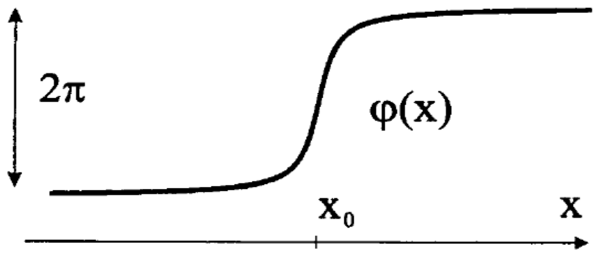

Fig. 2. Phase changing in long Josephson junction

3. Results and discussions

Using anharmonic current-phase relation (2) leads to the double sine-Gordon equation (Atanasova *et al*., 2010; Atanasova *et al*., 2011)

$$-\frac{d^2\phi}{dx^2} + \frac{1}{\lambda^2}(\sin\phi + \alpha\sin 2\phi) - \gamma = 0. \quad (6)$$

The magnitude γ is the external current, l is the semi-length of the junction. All the magnitudes are dimensionless. The boundary conditions for (6) have the form $\frac{d\phi}{dx}(\pm l) = h_e$.

Stability and bifurcations of static solutions $\phi(x, p)$, where $p = (l, \alpha, h_e, \gamma)$ are analyzed based on numerical solution of the corresponding Sturm-Louville problem (Atanasova *et al*., 2010; Atanasova *et al*., 2011)

$$-\frac{d^2\Psi}{dx^2} + q(x)\Psi = \lambda\Psi, \quad (7)$$

$$\Psi(\pm l) = 0, \; q(\phi) = \cos\phi + \frac{\alpha}{2}\cos 2\phi. \quad (8)$$

The minimal eigenvalue $\lambda_0(p) > 0$ corresponds to the stable solution. In case $\lambda_0(p) < 0$ solution $\phi(x,p)$ is unstable. The solutions of equation (6) are determined by a number of fluxons $N(p)$ which is defined as

$$N(p) = \frac{1}{2\pi l}\int_{-l}^{l}\phi(x)dx \quad . \tag{9}$$

In the harmonic case $\alpha = 0$ two trivial solutions $\phi = 0$ and $\phi = \pi$ of (7), (8) are known at $y = 0$ and $h_e = 0$, which are denoted by $M_0(N[M_0] = 0)$ and $M_\pi(N[M_\pi] = 1)$, respectively. Accounting of the second harmonic asin 2ϕ leads to the appearance of two additional solutions $\phi = \pm\arccos(-1/a)$ denoted as $M_{\pm ac}(N[M_{\pm ac}])$ are not integer numbers and depend on the value of second harmonic). Stability properties of trivial solutions in dependence on parameters are considered in (Atanasova *et al*., 2010; Atanasova *et al*., 2011). It was shown the deformation of the Meissner solution M_0. Inclusion of anharmonicity parameter in current-phase relation changes the shape and stability properties phase distribution in the long Josephson junction.

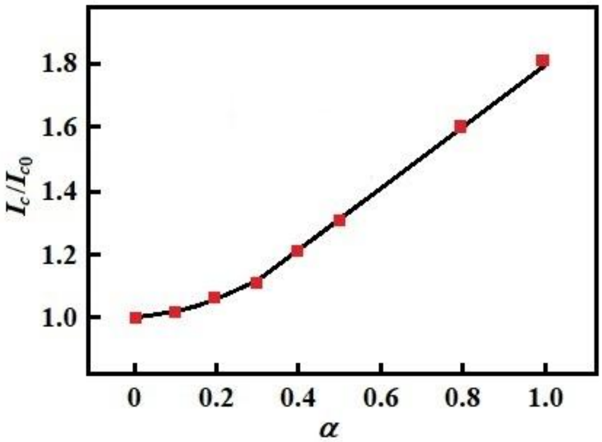

Fig. 3. Effective critical current of junction with anharmonic current-phase relation

As shown in (Kulikov, 2019; Askerzade *et al*., 2020) the presence of the second harmonic in the current-phase relation in Eq. (2) leads to the renormalization of critical current I_{c0}. In calculations, we use an analytical solution for the maximum point of the function $f_\alpha(\phi)$ *(2)* similar to (Goldobin *et al*., 2007). Calculation leads to the expression for the renormalized critical current at small anharmonicity parameter α

$$\frac{I_c}{I_{c0}} \approx 1 + 2\alpha^2 \ . \tag{10}$$

The effective critical current I_c of a junction with an anharmonic current-phase relation as a function of the amplitude of the second term α (see Eqs. (2)) is presented in Fig. 3. As you can see, with the increase of this amplitude, the effective critical current I_c has also increased. Quadratic behavior at small α (Eq. (2)) is converted to linear dependence at high values of anharmonicity parameter α. In our opinion this effective critical current approach can be adopted for the analysis of the Josephson penetration depth in long junctions. Recalculated Josephson penetration depth λ (Eq. (4), taking into account critical current renormalization (10)) is presented in Fig 4. It is clear that the Josephson penetration depth λ decreases with increasing anharmonicity parameter α. Such a result can be explained by the decrease of the Josephson junction inductance with an increase of critical current I_c and as a result of a weakening of inertial properties. It is useful to note that obtained results in qualitative agreement with results of Refs. (Atanasova *et al*, 2010; Atanasova *et al*., 2011).

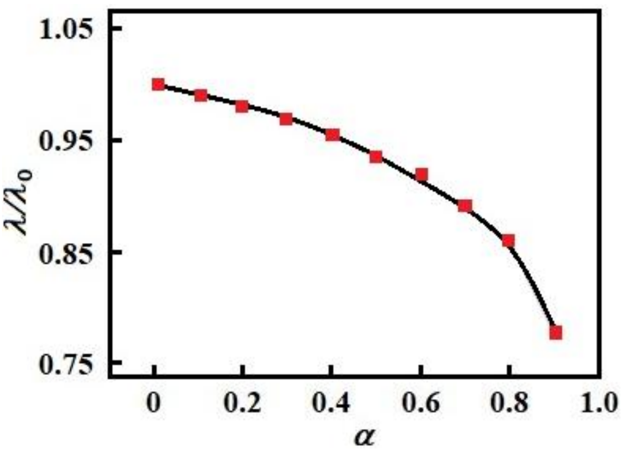

Fig. 4. Josephson penetration depth in long junction with anharmonic current-phase relation.

4. Conclusion

Thus, in this study, the long Josephson junction model with the second harmonic in current-phase relation is considered. The influence of the second harmonic on the stability of magnetic flux distributions for main solutions is analyzed. The effect of the second harmonic on Josephson penetration depth was calculated using numerical results for effective critical current.

Acknowledgements

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